NONCLASSICAL PROPERTIES OF SINGLE-PHOTON-ADDED TWO-MODE SU(1,1) COHERENT STATES

MAI THI LY - TRUONG MINH DUC
University of Education, Hue University

Abstracts: We study the nonclassical properties of single-photon-added two-mode SU(1,1) coherent states. We derive the analytic expressions of the sum squeezing, the difference squeezing, the higher-order squeezing, the higher-order antibunching and the violation of the Cauchy– Schwarz inequality. We show that in such states, squeezing appears in the sum squeezing and in the higher-order squeezing but not in the difference squeezing. We also show that these states not only exhibit higher-order antibunching to all orders but also completely violate the Cauchy – Schwarz inequality. As expected, when adding a photon to two modes of two-mode SU(1,1) coherent states, the degree of sum squeezing and Hillery higher-order squeezing become bigger but the degree of higher-order antibunching and violation of the Cauchy-Schwarz inequality become smaller.

Keywords: Single-photon-added two mode SU(1,1) coherent states, nonclassical properties.

1 INTRODUCTION

Recently in quantum optics there has been much interest in the photon-added coherent states. In particular, for the photon-added two-mode states, strong correlations between the modes are responsible for many nonclassical effects including sub-Poissonian statistics, squeezing and violation of the Cauchy– Schwarz inequality that is the premise for various applications in quantum optics, quantum information and quantum computation. The photon-added coherent states was first introduced by Agarwal and Tara [1] in 1991. Many states can be become to the photon-added coherent states by adding photons to single-mode or double-mode feilds of this state.

Journal of Science and Education, University of Education, Hue University
ISSN 1859-1612, No 02(42)/2017: pp. 15-28
Received: 01/10/2016; Revised: 15/10/2016; Accepted: 28/10/2016

Relied upon it, we adding a photon to two modes a and b of the two-mode SU(1,1) coherent states defined by Perelomov [2] in order to have a new state that is single-photon-added two-mode SU(1,1) coherent states as

$$\begin{aligned} |\psi\rangle_{ab} &= \mathcal{N}(\hat{a}^\dagger + \hat{b}^\dagger)|\varphi\rangle_{ab} \\ &= \mathcal{N}(\hat{a}^\dagger + \hat{b}^\dagger)(1-|\xi|^2)^{\dfrac{1+q}{2}} \sum_{n=0}^{\infty} \left[\frac{(n+q)!}{n!q!}\right]^{1/2} \xi^n |n+q, n\rangle_{ab}, \end{aligned} \tag{1}$$

where $\mathcal{N}$ is a normalization factor

$$\mathcal{N} = \left[2 + (1-|\xi|^2)^{1+q} \sum_{n=0}^{\infty} \frac{(n+q)!}{n!q!} |\xi|^{2n}(2n+q)\right]^{-\frac{1}{2}}, \tag{2}$$

$\hat{a}(\hat{a}^\dagger)$ and $\hat{b}(\hat{b}^\dagger)$ represent the annihilation (creation) operators of mode a and b, $|\varphi\rangle_{ab}$ is the two-mode SU(1,1) coherent states, $\alpha = -\frac{1}{2}\,\theta \exp(-i\varphi)$, $\xi = -\tanh\left(\frac{\theta}{2}\right) \exp(-i\varphi)$, $|n+q, n\rangle_{ab}$ is a Fock states containing $n+q$ photons in mode a and n photons in mode b, q is the degeneracy parameter or the difference between the photon numbers of modes a and b. Two-mode SU(1,1) coherent states have strong nonclassical properties such as sum squeezing, higher-order antibunching and violation of the Cauchy-Schwarz inequality [3]. In this paper, we consider how a photon addition affects on nonclassical properties of two-mode SU(1,1) coherent states by studying nonclassical properties of single-photon-added two-mode SU(1,1) coherent states. We investigate the degree of sum squeezing, different squeezing, Hillery higher-order squeezing, Hillery higher-order antibunching and the violation of the Cauchy-Schwarz inequality of single-photon-added two-mode SU(1,1) coherent states and compare with the degree of sum squeezing of two-mode SU(1,1) coherent states.

2 SUM SQUEEZING

The sum squeezing property definition was firstly given by Hillery [4] in 1989. A state is called a sum squeezed state if satisfying inequality below

$$S = \langle \hat{V}_\phi^2 \rangle - \langle \hat{V}_\phi \rangle^2 - \frac{1}{4}\langle \hat{n}_a + \hat{n}_b + 1 \rangle. \tag{3}$$

The sum squeezing exists when $S < 0$ and if the sum squeezing parameter S is more negative, that state will be more strongly squeezed. In two-mode systems, after the expansion, the sum squeezing parameter is written as

$$S = \frac{1}{4}\langle (e^{i\phi}\hat{a}^\dagger\hat{b}^\dagger)^2 + (e^{-i\phi}\hat{a}\hat{b})^2 + 2\hat{a}^\dagger\hat{b}^\dagger\hat{a}\hat{b} \rangle - \frac{1}{4}\left\{\langle e^{i\phi}\hat{a}^\dagger\hat{b}^\dagger\rangle + \langle e^{-i\phi}\hat{a}\hat{b}\rangle\right\}^2. \tag{4}$$

For the single-photon-added two-mode SU(1,1) coherent states, we have the sum squeezing parameter S:

$$
\begin{aligned}
S = &\frac{1}{2}\cos 2\gamma\left[2+(1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n} r\ (2n+q)\right]^{-\frac{1}{2}}\\
&\times\left[2+(1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q+2)!}{(n+2)!q!}\tanh^{2n+4} r\ (2n+q+4)\right]^{-\frac{1}{2}}\\
&\times(1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n+2} r\ (2n+q+1)(n+q+1)(n+q+2)\\
&+\frac{1}{2}\left[2+(1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n} r\ (2n+q)\right]^{-1}\\
&\times(1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n} r\\
&\times\left[(2n+q-2)n(n+q)+(6n+1)(n+q)+n\right]\\
&-\left\{\cos\gamma\ \left[2+(1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n} r\ (2n+q)\right]^{-\frac{1}{2}}\right.\\
&\times\left[2+(1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q+1)!}{(n+1)!q!}\tanh^{2n+2} r\ (2n+q+2)\right]^{-\frac{1}{2}}\\
&\left.\times(1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n+1} r\ (2n+q+4)(n+q+1)\right\}^2,
\end{aligned}
\tag{5}
$$

where $\phi+\varphi=\gamma, \theta=2r$ and $r\geq 0$. Based on the Eq. (5) and the sum squeezing parameter of two-mode SU(1,1) coherent states [3], we consider the dependence of the degree of sum squeezing of both two-mode SU(1,1) coherent states and single-photon-added two-mode SU(1,1) coherent states on amplitude squeezing r and degeneracy parameter q where $\gamma=\pi/2$. Fig. 1 shows that $S<0$ in every amplitude squeezing r and degeneracy parameter q so two these states exhibit sum squeezing where $\gamma=\pi/2$; squeezing becomes stronger when amplitude squeezing r and degeneracy parameter q are increased. At the same value of amplitude squeezing r and degeneracy parameter q, single-photon-added two-mode SU(1,1) coherent states are more sum squeezing than two-mode SU(1,1) coherent states. Therefore, we can conclude that the degree of sum squeezing becomes bigger when adding a photon to two modes of two-mode SU(1,1) coherent states.

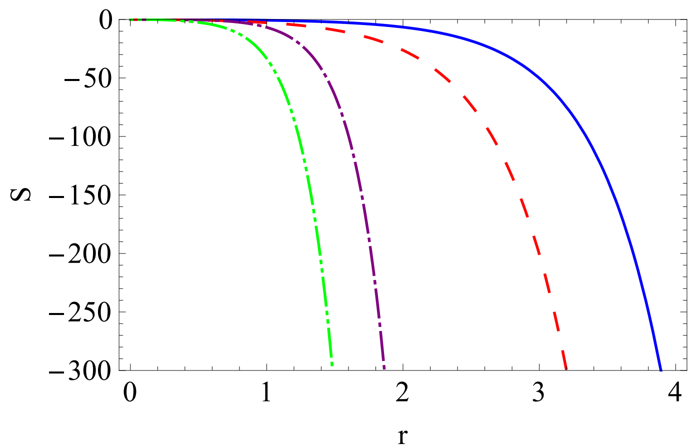


Fig. 1: The dependence of S of two-mode SU(1,1) coherent states where $q = 0$ (the solid curve), $q = 3$ (the dashed curve) and single-photon-added two-mode SU(1,1) coherent states where $q = 0$ (the dot–dashed curve), $q = 3$ (the double-dot-dashed curve) on amplitude squeezing r with $\gamma = \pi/2$.

3 DIFFERENCE SQUEEZING

According to Hillery [4], a state satisfying inequality below is called a difference squeezed state

$$D = \langle \hat{W}_\phi^2 \rangle - \langle \hat{W}_\phi \rangle^2 - \frac{1}{4}\langle \hat{n}_a - \hat{n}_b \rangle. \tag{6}$$

The difference squeezing exists when $D < 0$. If the difference squeezing parameter D is more negative, that state will be more strongly squeezed. In two-mode systems, after the expansion, we have the difference squeezing parameter D:

$$\begin{aligned} D =& \frac{1}{4}\langle (e^{i\phi}\hat{a}\hat{b}^\dagger)^2 + (e^{-i\phi}\hat{a}^\dagger\hat{b})^2 + \hat{a}\hat{b}^\dagger\hat{a}^\dagger\hat{b} + \hat{a}^\dagger\hat{b}\hat{a}\hat{b}^\dagger \rangle \\ & - \frac{1}{4}\left\{ \langle e^{i\phi}\hat{a}\hat{b}^\dagger \rangle + \langle e^{-i\phi}\hat{a}^\dagger\hat{b} \rangle \right\}^2 - \frac{1}{4}\langle \hat{n}_a - \hat{n}_b \rangle. \end{aligned} \tag{7}$$

For the single-photon-added two-mode SU(1,1) coherent states, we have the parameter D:

$$\begin{aligned} D = \frac{1}{4}&\left[2 + (1 - \tanh^2 r\ )^{1+q} \sum_{n=0}^{\infty} \frac{(n+q)!}{n!q!} \tanh^{2n} r\ (2n+q) \right]^{-1} \\ &\times \left\{ 2 + (1 - \tanh^2 r\ )^{1+q} \sum_{n=0}^{\infty} \frac{(n+q)!}{n!q!} \tanh^{2n} r \right. \\ &\times \left. \left[2n(n+q)(2n+q-2) + (15n+q)(n+q) + n(n+4) + 6n + 5q \right] \right\} \end{aligned}$$

$$
\begin{aligned}
&-\left\{\cos\phi\left[2+(1-\tanh^2 r\ )^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n} r\ (2n+q)\right]^{-1}\right.\\
&\times\left.\left(1+(1-\tanh^2 r\ )^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n} r\ \left[(n+1)(n+q)+n\right]\right)\right\}^2\\
&-\frac{1}{4}\left[2+(1-\tanh^2 r\ )^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n} r\ (2n+q)\right]^{-1}\\
&\times(1-\tanh^2 r\ )^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n} r\ q(2n+q+3),
\end{aligned}
\tag{8}
$$

where $\phi+\varphi=\gamma, \theta=2r$ and $r\geq 0$. Based on the difference squeezing parameter

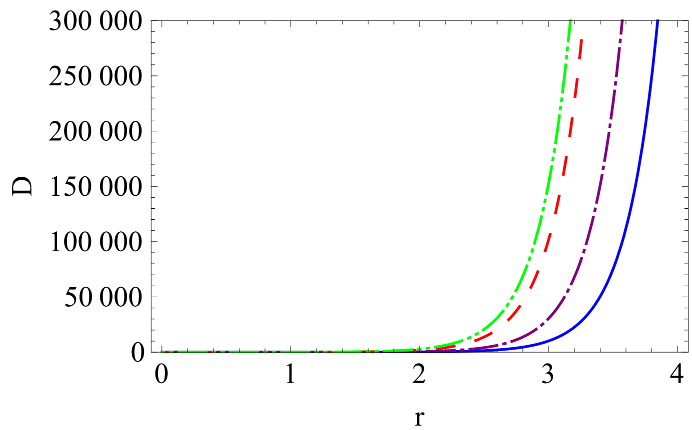


Fig. 2: The dependence of D of two-mode SU(1,1) coherent states where $q=0$ (the solid curve), $q=3$ (the dashed curve) and single-photon-added two-mode SU(1,1) coherent states where $q=0$ (the dot–dashed curve), $q=3$ (the double-dot-dashed curve) on amplitude squeezing r with $\phi=0$.

D of the single-photon-added two-mode SU(1,1) coherent states in Eq. (8) and the difference squeezing parameter of two-mode SU(1,1) coherent states [3], we consider the degree of difference squeezing of both two-mode SU(1,1) coherent states and single-photon-added two-mode SU(1,1) coherent states. In Fig. 2 we can see that $D>0$ in every amplitude squeezing r and degeneracy parameter q so that two these states do not exhibit difference squeezing in every values of ϕ. Thus, when adding a photon to two modes of two-mode SU(1,1) coherent states that still does not exist difference squeezing property.

4 HILLERY HIGHER-ORDER SQUEEZING

The Hillery higher-order squeezing was firstly introduced by Hillery [5]. In higher-order squeezing case, the squeezing associated with the two-mode quadrature operators

$$\hat{X}_{ab,k}(\phi) = \frac{1}{2\sqrt{2}}\big[(\hat{a}+\hat{b})^k e^{-i\phi} + (\hat{a}^\dagger + \hat{b}^\dagger)^k e^{i\phi}\big], k = 1, 2, 3, \ldots \tag{9}$$

The Hillery higher-order squeezing parameter is

$$H_k(\phi) = \langle(\hat{X}_{ab,k}(\phi))^2\rangle - \langle\hat{X}_{ab,k}(\phi)\rangle^2 - \frac{1}{8}\big|\langle\hat{F}_{ab}(k)\rangle\big|, \tag{10}$$

with $\hat{F}_{ab}(k) = \big[(\hat{a}+\hat{b})^k, (\hat{a}^\dagger+\hat{b}^\dagger)^k\big]$. The condition for a state to exists Hillery higher-order squeezing is $H_k(\phi) < 0$. When the parameter $H_k(\phi)$ is more nagative, that state will be more strongly Hillery higher-order squeezed. In case of $k = 2$, for the single-photon-added two-mode SU(1,1) coherent states, we have the Hillery second-order squeezing parameter $H_2(\phi)$:

$$\begin{aligned}
H_2(\phi) = \frac{5}{2}\ \cos2\gamma & \left[2 + (1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n} r\ (2n+q)\right]^{-\frac{1}{2}} \\
& \times \left[2 + (1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q+2)!}{(n+2)!q!}\tanh^{2n+4} r\ (2n+q+4)\right]^{-\frac{1}{2}} \\
& \times (1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n+2} r\ (2n+q+6)(n+q+1)(n+q+2) \\
& + \left[2 + (1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n} r\ (2n+q)\right]^{-1} \\
& \times (1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n} r \\
& \times \left\{\frac{1}{4}\big[(2n+q-2)(n+q)(n+q-1) + (6n+6q-2)(n+q)\right. \\
& + (2n+q-2)n(n-1) + (6n-2)n\big] + \big[(2n+q-1)(n+q)(n+1) \\
& \left. + (3n+2q)(n+1) + (2n+q-2)n(n+q) + (6n+1)(n+q) + n\big]\right\} \\
& - \frac{1}{2}\left\{\cos\gamma\left[2 + (1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n} r\ (2n+q)\right]^{-\frac{1}{2}}\right. \\
& \times \left[2 + (1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q+1)!}{(n+1)!q!}\tanh^{2n+2} r\ (2n+q+2)\right]^{-\frac{1}{2}}
\end{aligned}$$

$$\times (1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n+1}r$$
$$\times \left[3(n+q)(n+q+1)+3n(n+q+1)+12(n+q+1)\right]\Big\}^2, \tag{11}$$

where $\phi+\varphi=\gamma, \theta=2r$ and $r\geq 0$. From Eq. (11) and the Hillery second-order

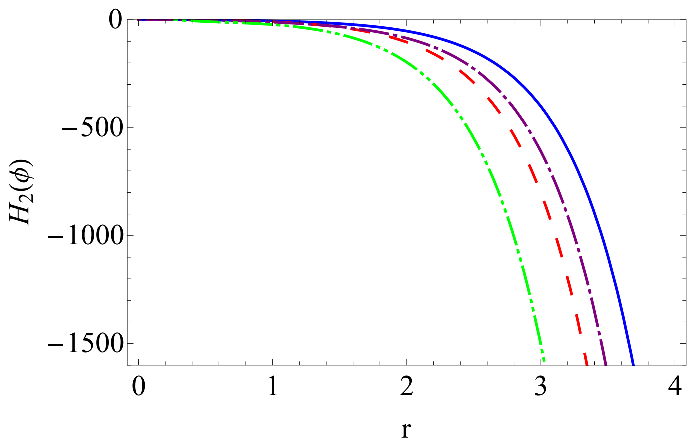


Fig. 3: The dependence of Hillery second-order squeezing $H_2(\phi)$ of two-mode SU(1,1) coherent states where $q=0$ (the solid curve), $q=3$ (the dashed curve) and single-photon-added two-mode SU(1,1) coherent states where $q=0$ (the dot–dashed curve), $q=3$ (the double-dot-dashed curve) on amplitude squeezing r with $\gamma=\pi/2$.

squeezing parameter of $H_2(\phi)$ of two-mode SU(1,1) coherent states, we can investigate the Hillery second-order squeezing degree of both two-mode SU(1,1) coherent states and single-photon-added two-mode SU(1,1) coherent states in every amplitude squeezing r. Fig. 3 shows that $H_2(\phi)<0$ in every amplitude squeezing r so two these states exhibit Hillery second-order squeezing where $\gamma=\pi/2$. When r and q are increased, the degree of Hillery second-order squeezing becomes stronger. At the same value of r and q, single-photon-added two-mode SU(1,1) coherent states are more Hillery second-order squeezing than two-mode SU(1,1) coherent states.

In case of $k=3$, for the single-photon-added two-mode SU(1,1) coherent states, the Hillery third-order squeezing parameter $H_3(\phi)$ is:

$$H_3(\phi)=-\frac{35}{4}\cos\beta\left[2+(1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n}r\ (2n+q)\right]^{-\frac{1}{2}}$$
$$\times\left[2+(1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q+3)!}{(n+3)!q!}\tanh^{2n+6}r\ (2n+q+6)\right]^{-\frac{1}{2}}$$

$$\begin{aligned}
&\times (1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n+3}r\\
&\times (2n+q+8)(n+q+1)(n+q+2)(n+q+3)\\
&+\frac{1}{8}\left[2+(1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n}r\ (2n+q)\right]^{-1}\\
&\times (1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n}r\\
&\times \Big[2(2n+q+5)n(n-1)(n-2)\\
&+2(2n+q+5)(n+q)(n+q-1)(n+q-2)\\
&+12(n+1)(n+q+1)(n+q)(n+q-1)\\
&+12(n+1)(n+q+1)n(n-1)+36(n+q)(n+q-1)\\
&+36n(n-1)+144n(n+q)+36(n+q+1)(n+1)(n+q)n\\
&+18(2n+q-2)(2n+q+5)n(n+q)\Big],
\end{aligned}\tag{12}$$

where $2\phi+3\varphi=\beta, \theta=2r$ and $r\geq 0$. Based on Eq.(12) and the Hillery third-order

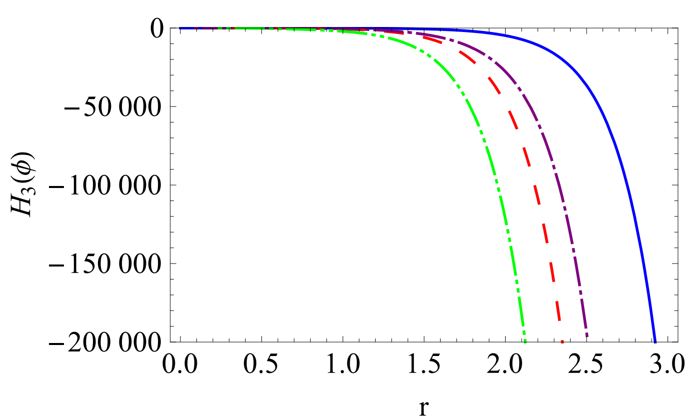


Fig. 4: The dependence of Hillery third-order squeezing $H_3(\phi)$ of two-mode SU(1,1) coherent states where $q=0$ (the solid curve), $q=3$ (the dashed curve) and single-photon-added two-mode SU(1,1) coherent states where $q=0$ (the dot–dashed curve), $q=3$ (the double-dot-dashed curve) on amplitude squeezing r with $\beta=0$.

squeezing parameter of $H_3(\phi)$ of two-mode SU(1,1) coherent states, we consider the Hillery third-order squeezing degree of two-mode SU(1,1) coherent states and single-photon-added two-mode SU(1,1) coherent states in every amplitude squeezing r. In Fig. 4 we can see that $H_3(\phi)<0$ in every amplitude r so two these states exist Hillery third-order squeezing where $\beta = 0$. The degree of Hillery third-order squeezing becomes stronger when r and q are increased. At the same value of r and q, single-photon-added two-mode SU(1,1) coherent states are more Hillery third-order squeezing than two-mode SU(1,1) coherent states.

From two cases of k that we have just considered, we can conclude that single-photon-added two-mode SU(1,1) coherent states exhibit Hillery higher-order squeezing in every amplitude squeezing r with suitable values of γ and β. Single-photon-added two-mode SU(1,1) coherent states are more Hillery higher-order squeezing than two-mode SU(1,1) coherent states at the same value of r and q. Therefore, when adding a photon to two modes of two-mode SU(1,1) coherent states, the degree of Hillery higher-order becomes stronger.

5 HIGHER-ORDER ANTIBUNCHING

The Higher-order antibunching property in two-mode case can be studied using the criterion introduced by Lee [6] in 1989. According to Lee, the criterion for the existence of intermode photon antibunching in a two-mode radiation field is defined via the coefficient $R_{ab}(l,p)$ as

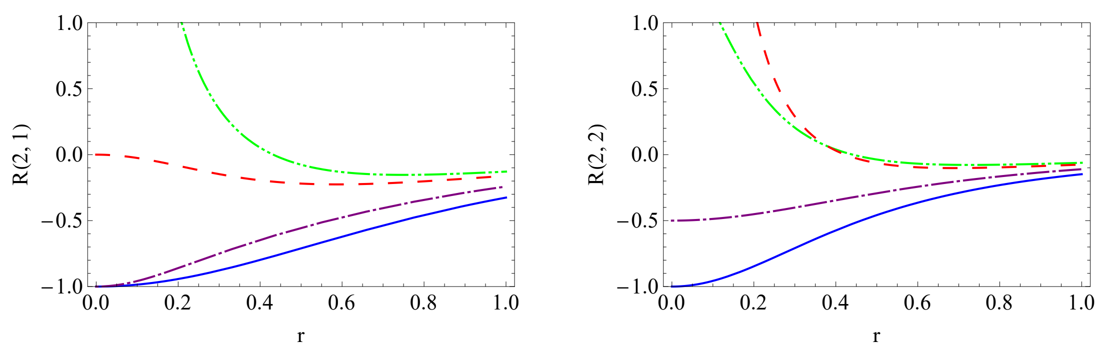
$$R_{ab}(l,p) = \frac{\langle \hat{n}_a^{(l+1)} \hat{n}_b^{(p-1)} \rangle + \langle \hat{n}_a^{(p-1)} \hat{n}_b^{(l+1)} \rangle}{\langle \hat{n}_a^{(l)} \hat{n}_b^{(p)} \rangle + \langle \hat{n}_a^{(p)} \hat{n}_b^{(l)} \rangle} - 1 < 0, \tag{13}$$

where $l \geq p > 0$ and $\langle \hat{n}_a^{(l)} \rangle = \langle \hat{a}^{\dagger l} \hat{a}^l \rangle$, $\langle \hat{n}_b^{(l)} \rangle = \langle \hat{b}^{\dagger l} \hat{b}^l \rangle$. Thus, the degree of antibunching becomes stronger when the coefficient $R_{ab}(l,p)$ is more negative. For the single-photon-added two-mode SU(1,1) coherent states, the coefficient $R_{ab}(l,p)$ is

$$\begin{aligned}
R_{ab}(l,p) = &\left[2 + (1-\tanh^2 r)^{1+q} \sum_{n=0}^{\infty} \frac{(n+q)!}{n!q!} \tanh^{2n} r \ (2n+q)\right]^{-1} \\
&\times (1-\tanh^2 r)^{1+q} \sum_{n=0}^{\infty} \frac{(n+q)!}{n!q!} \tanh^{2n} r \\
&\times \left[\frac{(n+q)!n!}{(n+q-l-2)!(n-p+1)!} + \frac{(n+q)!n!}{(n+q-l-1)!(n-p)!} \right. \\
&+ \frac{(n+q)!n!}{(n+q-p)!(n-l-1)!} + \frac{(n+q)!n!}{(n+q-p+1)!(n-l-2)!} \\
&+ \frac{2(l+p+1)(n+q)!n!}{(n+q-l-1)!(n-p+1)!} + \frac{2(l+p+1)(n+q)!n!}{(n+q-p+1)!(n-l-1)!}
\end{aligned}$$

$$
\begin{aligned}
&+\frac{(p-1)^2(n+q)!n!}{(n+q-l-1)!(n-p+2)!}+\frac{(p-1)^2(n+q)!n!}{(n+q-p+2)!(n-l-1)!}\\
&+\frac{(l+1)^2(n+q)!n!}{(n+q-l)!(n-p+1)!}+\frac{(l+1)^2(n+q)!n!}{(n+q-p+1)!(n-l)!}\Bigg]\\
&\times\left\{\left[2+(1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n}r\ (2n+q)\right]^{-1}\right.\\
&\times(1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n}r\\
&\times\left[\frac{(n+q)!n!}{(n+q-l-1)!(n-p)!}+\frac{(n+q)!n!}{(n+q-l)!(n-p-1)!}\right.\\
&+\frac{(n+q)!n!}{(n+q-p-1)!(n-l)!}+\frac{(n+q)!n!}{(n+q-p)!(n-l-1)!}\\
&+\frac{2(l+p+1)(n+q)!n!}{(n+q-l)!(n-p)!}+\frac{2(l+p+1)(n+q)!n!}{(n+q-p)!(n-l)!}\\
&+\frac{p^2(n+q)!n!}{(n+q-l)!(n-p+1)!}+\frac{p^2(n+q)!n!}{(n+q-p+1)!(n-l)!}\\
&\left.\left.+\frac{l^2(n+q)!n!}{(n+q-l+1)!(n-p)!}+\frac{l^2(n+q)!n!}{(n+q-p)!(n-l+1)!}\right]\right\}^{-1}-1,
\end{aligned}
\tag{14}
$$

where $\xi=-\tanh\left(\frac{\theta}{2}\right)\ \exp(-i\varphi)$, $\theta=2r$ and $r\geq 0$. Based on the higher-order



(a) (b)

Fig. 5: The dependence of (a) $R_{ab}(2,1)$, (b) $R_{ab}(2,2)$ of two-mode SU(1,1) coherent states where $q=0$ (the solid curve), $q=2$ (the dashed curve) and single-photon-added two-mode SU(1,1) coherent states where $q=0$ (the dot–dashed curve), $q=2$ (the double-dot-dashed curve) on amplitude squeezing r.

antibunching parameter of two-mode SU(1,1) coherent states [3] and the higher-order antibunching parameter of single-photon-added two-mode SU(1,1) coherent states in Eq. (13), we investigate the higher-order antibunching degree of both these states in every amplitude squeezing r and degeneracy parameter q in some case of (l,p) such as $R_{ab}(1,1)$, $R_{ab}(2,1)$, $R_{ab}(2,2)$, $R_{ab}(3,1)$, $R_{ab}(3,2)$, $R_{ab}(3,3)$, $R_{ab}(4,3)$. In Fig. 5 we plot two different cases of (l,p) of two-mode SU(1,1) coherent states and single-photon-added two-mode SU(1,1) coherent states. We find that the higher-order antibunching exists in all cases. With each particular value of (l,p), there exists a correspondence degree of antibunching. In both two-mode SU(1,1) coherent states and single-photon-added two-mode SU(1,1) coherent states, antibunching becomes stronger when the amplitude squeezing r and the difference between the photon numbers of modes a and b decreases. At the same value of r and q, two-mode SU(1,1) coherent states are more higher-order antibunching than single-photon-added two-mode SU(1,1) coherent states. Thus, when adding a photon to both modes of two-mode SU(1,1) coherent states, the higher-order antibunching degree becomes smaller.

6 VIOLATIONS OF THE CAUCHY-SCHWARZ INEQUALITY

The Cauchy– Schwarz inequality is given as [7]

$$I = \frac{\left[\langle \hat{a}^{\dagger 2}\hat{a}^2\rangle\langle \hat{b}^{\dagger 2}\hat{b}^2\rangle\right]^{1/2}}{|\langle \hat{a}^{\dagger}\hat{b}^{\dagger}\hat{b}\hat{a}\rangle|} - 1 \geq 0. \tag{15}$$

The Cauchy– Schwarz inequality is violated that mean [8]

$$I = \frac{\left[\langle \hat{a}^{\dagger 2}\hat{a}^2\rangle\langle \hat{b}^{\dagger 2}\hat{b}^2\rangle\right]^{1/2}}{|\langle \hat{a}^{\dagger}\hat{b}^{\dagger}\hat{b}\hat{a}\rangle|} - 1 < 0 \tag{16}$$

and the Cauchy– Schwarz inequality is more strongly violated when the coefficient I is more negative. For the single-photon-added two-mode SU(1,1) coherent states, we have the coefficient I is

$$\begin{aligned}
I = &\left[2 + (1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n} r\ (2n+q)\right]^{-1} \\
&\times \Bigg\{(1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n} r \\
&\times \left[(2n+q+4)(n+q)(n+q-1) + 4(n+q)\right] \\
&\times (1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n} r\ \left[(2n+q+4)n(n-1) + 4n\right]\Bigg\}^{1/2} \\
&\times \left\{\left[2 + (1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n} r\ (2n+q)\right]^{-1}\right.
\end{aligned}$$

$$\times\,(1-\tanh^2 r)^{1+q}\sum_{n=0}^{\infty}\frac{(n+q)!}{n!q!}\tanh^{2n} r$$
$$\times\,\Big[n(n+q)(2n+q-2)+(6n+1)(n+q)+n\Big]\Big\}^{-1}-1, \tag{17}$$

where $\xi=-\tanh\left(\frac{\theta}{2}\right)\,\exp(-i\varphi)$, $\theta=2r$ and $r\geq 0$. Based on coefficient I of both

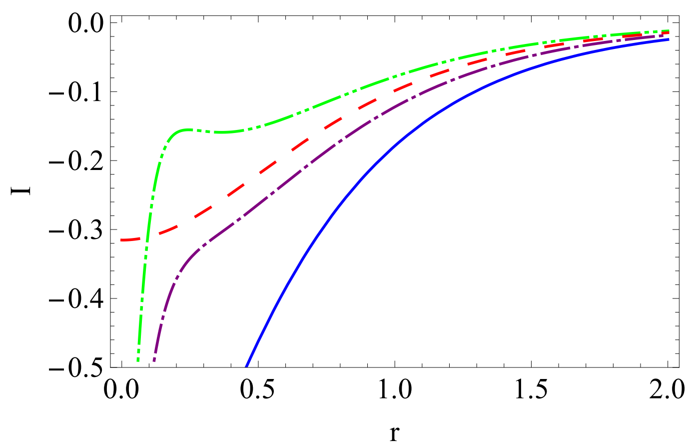


Fig. 6: The dependence of I of two-mode SU(1,1) coherent states where $q=1$ (the solid curve), $q=3$ (the dashed curve) and single-photon-added two-mode SU(1,1) coherent states where $q=1$ (the dot–dashed curve), $q=3$ (the double-dot-dashed curve) on amplitude squeezing r.

two-mode SU(1,1) coherent states [3] and single-photon-added two-mode SU(1,1) coherent states (17), we consider the degree of violation of the Cauchy-Schwarz inequality of these states. Fig. 6 shows that $I<0$ in every amplitude squeezing r and every degeneracy parameter q that means the Cauchy-Schwarz inequality is completely violated in both two-mode SU(1,1) coherent states and single-photon-added two-mode SU(1,1) coherent states. With each particular value of r and q, there exists a correspondence degree of the violation of the Cauchy-Schwarz inequality. The degree of the violation of the Cauchy-Schwarz inequality becomes stronger when r and q are decreased. At the same value of r and q, the degree of the violation of the Cauchy-Schwarz inequality of two-mode SU(1,1) coherent states is bigger than the degree of the violation of the Cauchy-Schwarz inequality of single-photon-added two-mode SU(1,1) coherent states. Thus, the degree of the violation of the Cauchy-Schwarz inequality of two-mode SU(1,1) coherent states becomes smaller when adding a photon to two modes of this state.

7. CONCLUSION

In summary, we have studied single-photon-added two-mode SU(1,1) coherent states in which we focused on their nonclassical properties and compared with nonclassical properties of two-mode SU(1,1) coherent states. In almost all cases the degree of nonclassical behavior of single-photon-added two-mode SU(1,1) coherent states depends strongly on the amplitude squeezing r and degeneracy parameter q. Single-photon-added two-mode SU(1,1) coherent states exhibit both two-mode sum squeezing and Hillery higher-order squeezing but do not exhibit two-mode difference squeezing. The sum squeezing and the Hillery higher-order squeezing only appear where suitable values of γ and β. Single-photon-added two-mode SU(1,1) coherent states also exist higher-order antibunching. With each particular value of (l, p), exsist a correspondence degree of antibunching. This state is not only exhibit higher-order squeezing and higher-order antibunching but also violate the Cauchy-Schwarz inequality. Throughout some investigation about nonclassical properties of two-mode SU(1,1) coherent states and single-photon-added two-mode SU(1,1) coherent states, we finally concluded that the degree of sum squeezing and Hillery higher-order squeezing become stronger but the degree of higher-order antibunching and the violation of the Cauchy-Schwarz inequality becomes weaker when adding a photon to two modes of two-mode SU(1,1) coherent states.

Acknowledgments

This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 103.01-2014.53.

REFERENCES

[1] Agarwal. G. S , Tara. K (1991), *Phys. Rev. A* **43**, pp. 492-497.

[2] Perelomov. A. M (1972), *Commun. Math. Phys.* **26**, pp. 222-236.

[3] Lê Đình Nhân, Trương Minh Đức (2015), *Tạp chí Khoa học và Giáo dục*, Trường Đại học Sư Phạm Huế, Số 01(33), trang 35-43.

[4] Hillery. M (1989) *Phys. Rev. A* **45**, pp. 3147-3155.

[5] Hillery. M(1987), *Opt. Commun. A* **62**, pp. 135-138;
Hillery. M (1987), *Phys. Rev. A* **36**, pp. 3796-3802.

[6] Lee. C. T (1989), *Phys. Rev. A* **41**, pp. 1569-1575.

[7] Loudon. R (1980), *Rep. Prog. Phys. A* **43**, pp. 913-949.

[8] Agarwal. G. S (1988), *J. Opt. Soc. Am B* **5**, pp. 1940-1947.